Availability Investigation of Free Space Optical Links with Time Diversity for Turbulence Channels Modeled with the K-Distribution

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Abstract. The terrestrial free space optical (FSO) communication systems, are attaining license free, high bandwidth access and security with low installation and operation cost. On the other hand, their main disadvantage is the continuous variations of the characteristics of the electromagnetic beam's propagation path, which is the atmosphere, causing performance mitigation on system's availability and performance. A very important phenomenon which induces these variations is the atmospheric turbulence. In order to minimize its influence on the characteristics of the FSO links, many techniques have been investigated and among them, the diversity techniques have attracted significant research interest. In this work, we investigate the availability, by means of their outage probability estimation, of FSO communication links which are using the time diversity configuration, over strong atmospheric turbulence channels modelled with the K-distribution. For this setup, we derive closed form mathematical expressions for the estimation of outage probability for various atmospheric turbulence strengths. Finally, we present numerical results for many cases with various turbulence strengths and time diversity characteristics.

Keywords: K-distribution, Diversity Techniques, Atmospheric Turbulence, Outage Probability, Wireless Optical Communication Systems.
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INTRODUCTION

The terrestrial outdoor wireless communication systems which are using the optical frequencies, i.e. the free space optical (FSO) communication systems, are combining many advantages such as, license free, high bandwidth access and security with low installation and operation cost, [1]-[4]. Obviously, the propagation path for the electromagnetic beam is the atmosphere. Thus, any change of its characteristics causes performance variations. One of them, is the atmospheric turbulence which represents a very significant availability and performance mitigation factor [1], [2], [4]-[10], affecting both, intensity and phase of the optical signal [11]. This phenomenon, for the case of a typical outdoor line-of-sight, point-to-point optical link causes even rapid fluctuations at the irradiance of the signal at the receiver's end. As a result, the optical channel is showing randomly time-varying characteristics due to the socalled scintillation effect, [1], [2], [5]-[8], which are decreasing the FSO system's availability and performance.

In order to counterbalance the influence of the turbulence effect on the availability and performance of FSO links, the diversity techniques have attracted significant attention by many research groups [11]-[18]. This technique is used in RF communication systems and most of the times is realized in space, in time or in wavelength. For the spatial diversity, [12]-[16], the optical communication system uses multiple transmitters and/or receivers at different places which are transmitting and receiving copies of the same part of the information signal. For the links with time diversity [17]-[19], there is only one trans–receiver pair, but the information signal is retransmitted more than once at different time slots. Finally, the systems which are using the wavelength diversity, [11], [19]-[21], use a composite transmitter and the signal is transmitted at the same time at different wavelengths towards a number of wavelength-selected receivers.

In this work we investigate the availability of FSO communication systems which are using the time diversity schemes over strong atmospheric turbulence channels modelled with the K-distribution [22]-[24]. For this point-to-point configuration, which consists of one transmitter and one receiver, in each side of the link [18], [19], we study its availability by means of the outage probability estimation for various atmospheric turbulence conditions. For the estimation of this metric we derive closed form mathematical expressions.

The rest of this paper is organized as follows: in section II we analyze the basic characteristic of the FSO link with time diversity and we present the channel model. In section III, we derive the mathematical expression for the estimation of the outage probability while, in section IV we present the corresponding numerical results and in section V we outline the conclusions.

SYSTEM AND CHANNEL MODEL

In an FSO link that uses time diversity schemes, the signal is transmitted in multiple copies, M, in different time slots. Thus, every copy is transmitted through the same spatial channel but with different turbulence characteristics each time due to the fact that it is a very rapidly varying phenomenon. As mentioned above, this system is using only one transmitter and one receiver in each side of the optical link. Hence, this procedure can be emulated as to send each copy in different receivers at the same time, i.e. similar as a system with one transmitter (single input) and multiple receivers (multiple outputs) - SIMO).

This link with time diversity can be described by using a binary input and continuous output with intensity modulation – direct detection (IM/DD) and the modulation we use is on-off keying (OOK). The channel is assumed to be memoryless, stationary with independent and identically distributed intensity (i.i.d) fast fading statistics with additive Gaussian noise (AWGN). Thus, as it is mentioned above, in order to model the FSO link with time diversity which is sending M times each part of the signal, we assume that the FSO link can be studied as a communication system with one transmitter and M receivers. Thus, the statistical channel model is given as [15], [18], [19]:

$$y_m = s_m x + n = \eta_m x I_m + n, \qquad m = 1, ..., M \tag{1}$$

where y_m represents the signal arriving at the receiver, s_m is the instantaneous gain and is equal to ηI_m where η is the effective photo-current conversion ratio of the receiver while I_m is the instantaneous normalized irradiance at the receiver, x is the modulated signal that is transmitted and takes the values "0" or "1" and finally, n is the AWGN with zero mean and variance $N_0/2$.

For the cases of strong turbulence conditions, the induced fading can be assumed as a random process that follows the K-distribution [15], [22]-[24]. The corresponding probability density function (pdf) is given as [23]:

$$f_{I_m}(I_m)=\frac{2(b_m)^{\frac{1+b_m}{2}}}{\Gamma(b_m)}I_m^{\frac{1+b_m}{2}-1}K_{b_m-1}\left(2\sqrt{b_mI_m}\right) \tag{2}$$

where $K_v(.)$ stands for the modified Bessel function of the second kind of order v, $\Gamma(.)$ is the gamma function while the parameter b_m is related to the effective number of discrete scatterers [22]-[24].

The corresponding cumulative distribution function (cdf) for the K-distribution is calculated by integrating (2) and concludes to the following expression [15], [23]:

$$F_{I_m}(I_m)=\frac{(b_mI_m)^{\frac{1+b_m}{2}}}{\Gamma(b_m)}G_{1,3}^{2,1}\left(b_mI_m\left|\begin{matrix}\frac{1-b_m}{2}\\ \frac{1-b_m}{2},\frac{b_m-1}{2},-\frac{1+b_m}{2}\end{matrix}\right.\right) \tag{3}$$

where $G_{p,q}^{m,n}[\cdot]$ stands for the Meijer G-function, [25].

Next, we define the instantaneous electrical signal-to-noise ratio (SNR) as $\xi_m=(\eta I_m)^2/N_0=s_m^2/N_0$, [10], [18], and the average electrical SNR as $\mu=(\eta E[I_m])^2/N_0$, [10], [18], [26], where $E[.]$ stands for the expected value of the normalized irradiance I_m. Using the expressions of ξ_m and μ_m in (2) and after a power transformation of I_m, the pdf of K distribution for ξ_m is obtained in the following form [15], [23]:

$$f_{\mu_m}(\mu_m)=\frac{(b_m)^{\frac{1+b_m}{2}}}{\Gamma(b_m)}\frac{\xi_m^{\frac{1+b_m}{4}-1}}{\mu_m^{\frac{1+b_m}{4}}}K_{b_m-1}\left(\sqrt[4]{\frac{16b_m\xi_m}{\mu_m}}\right) \tag{4}$$

while the corresponding cdf of K-distribution has the following form [15], [23]:

$$F_{\xi_m}(\xi_m)=\frac{(b_m)^{\frac{1+b_m}{2}}}{\Gamma(b_m)}\left(\frac{\xi_m}{\mu_m}\right)^{\frac{1+b_m}{4}}G_{1,3}^{2,1}\left(b_m\sqrt{\frac{\xi_m}{\mu_m}}\left|\begin{matrix}\frac{1-b_m}{2}\\ \frac{1-b_m}{2},\frac{b_m-1}{2},-\frac{1+b_m}{2}\end{matrix}\right.\right) \tag{5}$$

OUTAGE PROBABILITY OF THE FSO LINK WITH TIME DIVERSITY

The above cdf can conclude to a mathematical expression for the estimation of the outage probability of the FSO communication systems with time diversity, over turbulent channels modeled with the K distribution. This metric, represents the probability that the instantaneous electrical SNR falls below a critical threshold, ξ_{th}, which represents the receiver's sensitivity limit and thus, it is a particularly important parameter for systems' designing [10], [18], [19]. The outage probability for each one of the M copies of the signal is given as [10], [18], [19], [27]:

$$P_{out,m} = \Pr\left(\xi_m \le \xi_{th}\right) = F_{\xi_m}(\xi_{th}), \tag{6}$$

In this section we will derive the expression for the outage probability of an FSO link when the turbulence channel is modeled by K-distribution and the system uses time diversity. Due to the independence of outage probability of each copy of the M copies of the signal that will be transmitted, the outage probability of the FSO link can be estimated as [12], [18], [19]:

$$P_{out,M} = \Pi_{m=1}^{M} \Pr(\xi_m \le \xi_{th}) = \Pi_{m=1}^{M} F_{\xi}(\xi_{th}) \tag{7}$$

Thus, from (5) and (7) we conclude the following finite product for the estimation of the probability of error for the FSO time diversity scheme over strong turbulence conditions:

$$P_{out}(\xi_{th}) = \prod_{m=1}^{M}\left[\frac{b_m^{\frac{1+b_m}{2}}}{\Gamma(b_m)}\left(\frac{\xi_{th}}{\mu_m}\right)^{\frac{1+b_m}{4}} G_{1,3}^{2,1}\left(b\sqrt{\frac{\xi_{th}}{\mu_m}}\left|\begin{matrix}\frac{1-b_m}{2}\\ \frac{1-b_m}{2},\frac{b_m-1}{2},-\frac{1+b_m}{2}\end{matrix}\right.\right)\right] \tag{8}$$

The above expression, i.e. Eq. (8), can be further simplified by taking into account that the FSO link is using time diversity. Thus, in this scheme, as mentioned above, is using only one pair of trans-receiver in each side of the optical link and as a result, the spatial propagation path, for all the copies of the information signal, is the same. Thus, we can assume accurately, that the value of b_m, of Eqs (4) and (5), as well the average electrical SNR, μ_m, remain, practically, invariable for all the M transmitted copies [18], [19]. Hence, in (8) we can assume $b=b_1=b_2=\ldots=b_M$ and $\mu=\mu_1=\mu_2=\ldots=\mu_M$, as well, [18], [19].

Using the above assumptions the expression of Eq. (8) results to the following closed form mathematical expression for the estimation of FSO system's availability through the estimation of its outage probability for strong atmospheric turbulence channels modeled with the K-distribution:

$$P_{out}(\xi_{th}) = \left[\frac{b^{\frac{1+b}{2}}}{\Gamma(b)}\left(\frac{\xi_{th}}{\mu}\right)^{\frac{1+b}{4}} G_{1,3}^{2,1}\left(b\sqrt{\frac{\xi_{th}}{\mu}}\left|\begin{matrix}\frac{1-b}{2}\\ \frac{1-b}{2},\frac{b-1}{2},-\frac{1+b}{2}\end{matrix}\right.\right)\right]^{M} \tag{9}$$

It is important to mention here that the expression (8) can be used for many types of diversity, such as spatial, wavelength, etc, while the expression (9) stands for the time diversity scheme. Additionally, this expression, i.e. Eq. (9), estimates the total probability of outage of an FSO link with time diversity over strong turbulence conditions. It is worth mentioning here that this result, i.e. Eq. (9), generalizes the expression of the outage probability of the single FSO point-to-point link without diversity, obtained in [23].

NUMERICAL RESULTS

Using the mathematical expressions obtained in Eqs (8) and (9), the outage probability estimation of an FSO link with time diversity under strong atmospheric turbulence conditions modelled with the K distribution, is feasible. By studying quantitatively these expressions it is clear that this technique improves the system's availability without need of extra trans-receivers, in each side of the FSO links. On the other hand, due to the finite bandwidth of the channel and the one and only spatial propagation path, the performance of the link is decreasing, as shown in [18], [19], [28]. Thus, in order to design an FSO link with time diversity, in practice, we should choose the correct diversity strength, i.e. the value of *M*, according to the requirements of each FSO link.

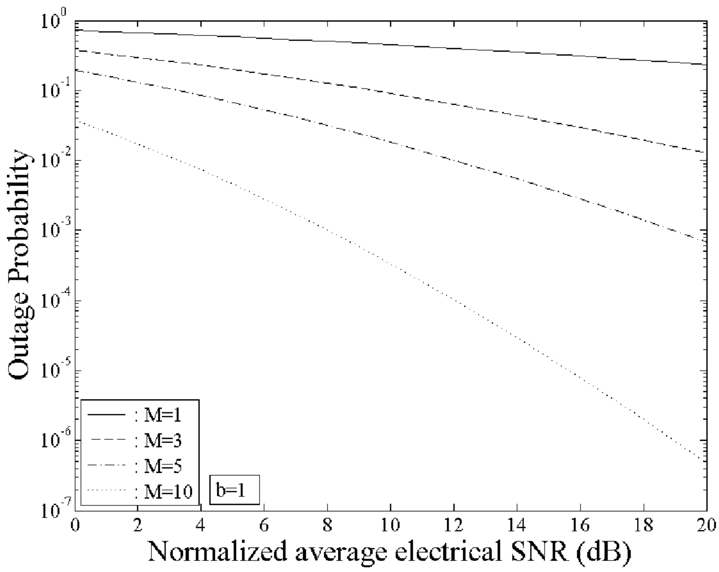

FIGURE 5. Outage probability, P_{out}, of an FSO link with time diversity, modeled with the K-distribution, versus the normalized average electrical SNR, μ/ξ_{th}, with *b*=1 and various values of time diversity parameter *M*.

In this section, we present the availability results for the FSO links with time diversity, using the extracted mathematical expressions of Eq. (9). The important parameters of the FSO link which are modelled with the K distribution, are the number of repetitions of the information signal, *M*, the normalized average electrical SNR at the receiver, μ/ξ_{th}, and the parameter *b*. For the parameter *b* we choose three values, i.e. 1, 7 and 15, where the smaller values correspond to stronger turbulence conditions. On the other hand, for the time diversity parameter, *M*, we consider the values 1, which corresponds to an FSO link without diversity, 3, 5 and 10.

In Figure (1) we present the results for the case of very strong turbulence conditions, i.e *b*=1. It is obvious that for the case without diversity, even the smaller value of outage probability, i.e for large value of the normalized average electrical SNR, is giving a very large value which is not acceptable for FSO communication links. Thus, the same link with strong, *M*=5, or stronger, *M*=10, time diversity achieves much better values for this probability metric. It is obvious that for larger values of *M*, the availability improvement would be better, but, as mentioned above, further increase of *M*, will result in significant decrease of the effective (practical) bit rate of the system [18], [19], [28].

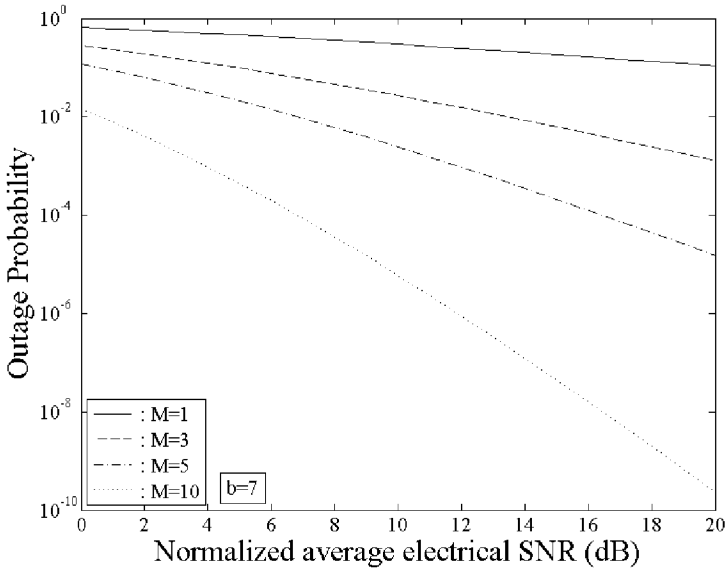

FIGURE 2. Outage probability, P_{out}, of an FSO link with time diversity, modeled with the K-distribution, versus the normalized average electrical SNR, μ/ξ_{th}, with *b*=7 and various values of time diversity parameter *M*.

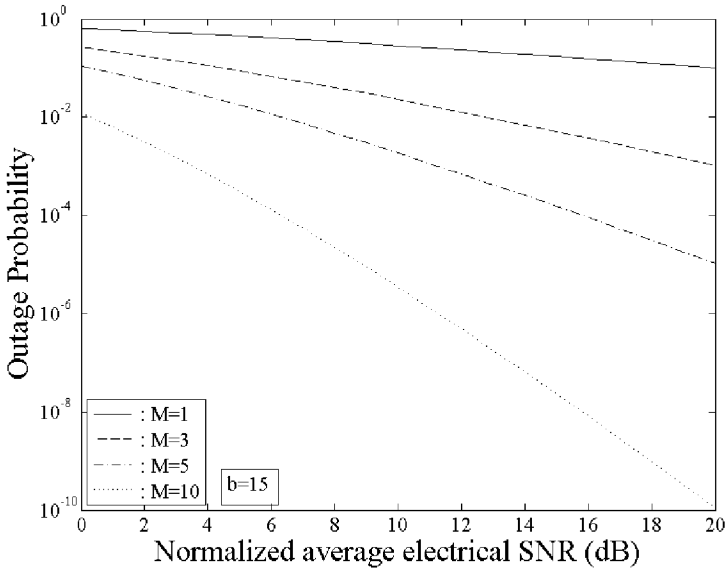

FIGURE 3. Outage probability, P_{out}, of an FSO link with time diversity, modeled with the K-distribution, versus the normalized average electrical SNR, μ/ξ_{th}, with *b*=15 and various values of time diversity parameter *M*.

The above conclusion, is clear for larger values of *b*, i.e. for weaker atmospheric turbulence conditions than those of Figure (1). Thus, in Figures (2) and (3) we present the corresponding results for *b*=7 and 15. In both cases, it is obvious that the values of outage probability, without time diversity, i.e. *M*=1, are smaller than those of the previous case with *b*=1, but they are still large and not acceptable for practical FSO links, even for large values of normalized average electrical SNR. This situation is much better when the time diversity scheme is used. As we can observe in Figures (2) and (3), for time diversity values, M=5 and M=10, the link can easily achieve very small values of this probability and thus the system can be easily characterized as one with very high availability.

CONCLUSIONS

In this work we investigated an FSO system which is using a time diversity scheme in order to increase its availability against atmospheric turbulence effect. More specifically, we investigate the capabilities of a terrestrial, outdoor, FSO link with time diversity, which is working under strong turbulence conditions modeled with the K-distribution. For this case we extract both, general and more specific, closed form mathematical expressions for the estimation of the link's availability through the estimation of outage probability. We prove that this technique can decrease significantly the system's probability of outage without need of extra trans-receivers in each side of the link. Moreover, in the section with the numerical results, using the above derived mathematical expressions, we show that the achieved improvement of the link's availability characteristics are significant and consequently, the whole system can be described as one with very high availability.

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