BLER of SIMO FSO Systems over Saturated Turbulence Channels and Pointing Errors

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Abstract. During the last years, terrestrial free space optical (FSO) systems have attracted great commercial and research interest as they offer license-free and very high bandwidth access characteristics with very low installation and operational cost. Their successful presence in demanding applications and networks, such as real-time Military Theater of Operations, can guarantee high performance and security for naval applications e.g. communications in shipyard between fixed or mobile transceivers. On the other hand, weather conditions and atmospheric turbulence affect significantly the propagation of the laser beam that transmits the information, deteriorating the performance of the FSO system. Additionally, another factor that decreases the performance of such systems is the pointing errors due to the misalignment between the receiver and the transmitter. Both turbulence and pointing errors are responsible for fast and intensive power fluctuations at the receiver i.e. the scintillation effect, so the optical channel can be investigated statistically and modelled accurately through various statistical distribution models depending on the effects' strength. Thus, for saturated turbulence, negative exponential statistical distribution can be used in order to model the channel precisely. To overcome the system's degradation that is caused by turbulence and pointing errors, various techniques have been proposed and used, with the receivers' diversity scheme being a very effective one. Thus, in this work, we investigate the effect of saturated turbulence conditions and pointing errors and how the deployment of receivers' diversity can effectively increase the performance of the system. The metric that will be investigated and is related to the performance and reliability of an FSO system is the block error rate (BLER). The specific quantity is a very significant one, especially for the very high data rate communication systems, such FSOs, because its outcome can specify the kind of coding scheme which should be used. Thus, novel, closed form mathematical expressions for the estimation of the average BLER of the system will be derived and the corresponding numerical results are presented. Furthermore, using the obtained simulation outcomes, the theoretical predictions of this work, will be verified.

Keywords: FSO communications, SIMO FSO systems, Negative exponential turbulent channel, pointing errors, spatial receiver's diversity, average BLER
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I. INTRODUCTION

Nowadays, the demand for faster and more efficient communication urges the field of telecommunications towards more modern systems. A technology with great potential, that is continuously attracting more attention is the free space optics (FSO) technology.The terrestrial FSO systems can provide very high data rates and almost unlimited bandwidth, due to the physical properties of light, while their equipment is characterized by low installation and upkeep cost and low energy consumption. They can, also, offer high security links, as a result of their line – of – sight (LoS) requirement.

However, the performance of an FSO system is affected by numerous phenomena that decrease their efficiency. A typical terrestrial FSO link consists of a laser or LED as the transmitter and a photodetector at the receiver's end. Because of the light beam propagating through the atmospheric medium, their operation depends significantly on the weather conditions. Adverse weather conditions, such as rain, snow, fog, etc. increase the absorption and scattering of the beam and hinder the stable and efficient communication between the two ends of the link. Another phenomenon, that originates from the stochastic nature of the atmosphere, is the atmospheric turbulence, which causes local variations of the refractive index of the atmosphere. These variations have a negative impact on the propagating light beam, leading to fast and random fluctuations in the irradiance of the received pulse, known as scintillations,[1]–[6].

Moreover, the FSO systems are point – to – point (PtP) links and their requirement for LoS is mandatory. Consequently, they are usually installed on top of high – rise buildings. Certain effects, such as thermal expansion, strong wind loads and weak earthquakes, can cause the sway of the buildings, resulting in misalignments of the beam from the center of the detector, also called pointing errors effect. The latter can cause, in turn, the partial collection of the transmitted power, which translates into fluctuations in the irradiance that reaches the detector and sever degradations to the performance of the communication link, [7]–[10].

A common technique used to counterbalance the degradations induced to the optical link is the receiver's diversity technique. In the case of spatial diversity, the signal is transmitted in several copies by one transmitter towards multiple receivers, creating a single – input – multiple – output (SIMO) system. Due to the fact that the same signal is detected by more than one receiver, the probability of bit errors appearing at the receiver's end is minimized and the reliability of the link is increased,[7], [11]–[14].

In the current work, the aim is to investigate the performance of a terrestrial SIMO FSO system, which operates under saturated turbulence conditions with additional pointing errors effect. In section II, the necessary statistical study of the channel will be made by analyzing the chosen statistical models and the combined probability density function (PDF) will be extracted. The combined PDF will, then, be used in section III to obtain a new closed – form formula for the average block error rate of the system, in order to evaluate its performance. Finally, in section IV, the corresponding numerical results will be presented.

II. SYSTEM AND CHANNEL MODEL

The SIMO FSO system studied in the current work is using a spatial diversity scheme, which means that the signal is transmitted from the laser or LED source (transmitter) and is detected by

K photodetectors (receivers) on the other end of the link. The optical beam is assumed to propagate horizontally between the transmitter and the receivers, through a turbulent atmospheric channel with additive white Gaussian noise (AWGN). It is, also, assumed that an intensity modulation / direct detection scheme is employed, while the channel is considered to be memoryless, stationary and ergodic. Under these assumptions, the FSO signal reaching each detector can take the form,[11]:

$$y_l = \eta_l x I_l + n_l, \qquad l = 1, 2, ..., K \tag{1}$$

where y_l is the signal arriving at the *l-th* receiver, η_l is the quantum efficiency of the detector, x is the modulated signal and n_l represents the AWGN with power spectral density σ_l^2. Parameter I_l is the normalized received irradiance and assuming that the optical pulse suffers from the independent effects of turbulence and pointing errors, it can be written as, [7], [15]:

$$I_l = I_{t,l} I_{p,l} \tag{2}$$

where $I_{t,l}$ and $I_{p,l}$ are the received irradiances at the *K-th* receiver, considering separately the degradations induced by turbulence and pointing errors, respectively.

During the propagation of the pulse, the signal is degraded by the atmospheric turbulence. In the case of strong and even saturated turbulence, the effect on the irradiance of the received pulse at the *K-th* receiver, $I_{t,l}$, can be statistically described by the negative exponential distribution, and the probability density function (PDF) is given as, [15]–[17]:

$$f_{I_{t,l}}\left(I_{t,l}\right) = \exp\left(-I_{t,l}\right) \tag{3}$$

Apart from turbulence, the other phenomenon that greatly affects the quality of the FSO link is the pointing error effect. Considering that the detector is equipped with a circular aperture of radius R and the beam reaching this aperture has a Gaussian intensity profile, [18], the irradiance collected by the detector can be approximated by the Gaussian form, [18]:

$$I_{p,l}\left(r_l\right) \approx A_{0,l} \exp\left(-\frac{2r_l^2}{w_{z,l,eq}^2}\right) \tag{4}$$

where r is the radial displacement of the footprint of the beam from the center of the detector's aperture after propagation distance z and $A_{0,l} = \left[erf\left(\upsilon_l\right)\right]^2$ is the fraction of the beam's power collected at $r_l = 0$, with υ_l given by $\upsilon_l = \left(\sqrt{\pi}R\right) / \left(\sqrt{2}w_{z,l}\right)$, while $w_{z,l}$ is the beam width and $w_{z,l,eq}$ is the equivalent beam width, defined as, [10], [18]:

$$w_{z,l,eq}^2 = w_{z,l}^2 \frac{\sqrt{\pi}erf\left(\upsilon_l\right)}{2\upsilon_l \exp\left(-\upsilon_l^2\right)} \tag{5}$$

If the vertical and horizontal parts of the displacement are modeled by independent identical Gaussian distributions, the radial displacement can be expressed by the Rayleigh distribution with PDF, [18]:

$$f_{r_l}\left(r_l\right)=\frac{r_l}{\sigma_{s,l}^2}\exp\left(-\frac{r_l^2}{2\sigma_{s,l}^2}\right) \tag{6}$$

for $r>0$, where σ_s^2 is the jitter variance at the receiver. Equations (4) and (6) lead to the PDF describing the normalized received irradiance, due to pointing errors, I_p as, [10], [15], [18]:

$$f_{I_{p,l}}\left(I_{p,l}\right)=\frac{g_l^2}{A_{0,l}^{g_l^2}}I_{p,l}^{g_l^2-1} \tag{7}$$

for $0\le I_{p,l}\le A_{0,l}$. In (7), $g_l=w_{z,l,eq}/2\sigma_{s,l}$ is the ratio between the equivalent beam and the displacement standard deviation and governs the intensity of the pointing error effect, with higher values corresponding to a weaker phenomenon.

The behavior of the FSO channel can be described by combining the PDFs (2) and (7), via the formula, [7], [15]:

$$f_{I_l}\left(I_l\right)=\int f_{I_l|I_{t,l}}\left(I_l\,|\,I_{t,l}\right)f_{I_{t,l}}\left(I_{t,l}\right)dI_{t,l} \tag{8}$$

where $f_{I_l|I_{t,l}}\left(I_l\,|\,I_{t,l}\right)$ is the conditional probability given the turbulence affected irradiance, I_t, and is calculated by, [7], [15]

$$f_{I_l|I_{t,l}}\left(I_l\,|\,I_{t,l}\right)=\frac{g_l^2}{A_{0,l}^{g_l^2}I_{t,l}}\left(\frac{I_l}{I_{t,l}}\right)^{g_l^2-1} \tag{9}$$

for $0\le I_l\le A_{0,l}I_{t,l}$. The combined PDF for the studied FSO channel is obtained from (2), (7) and (9) into (8) and takes the form, [15]:

$$f_{I_l}\left(I_l\right)=\frac{g_l^2}{A_{0,l}}G_{1,2}^{2,0}\left(\frac{I_l}{A_{0,l}}\middle|\begin{matrix}g_l^2\\ g_l^2-1,\quad 0\end{matrix}\right) \tag{10}$$

where $G_{u,v}^{m,n}(.)$ is the Meijer function, [19]. The above equation (10) can, also, be expressed in terms of the instantaneous received signal – to – noise ratio (SNR) of the pulse at the *K-th* receiver, γ_l, as:

$$f_{\gamma_l}\left(\gamma_l\right)=\frac{g_l^4\Gamma\left(g_l^2\right)}{2A_{0,l}\Gamma\left(1+g_l^2\right)}\left(\frac{1}{\xi_l\gamma_l}\right)^{1/2}G_{1,2}^{2,0}\left(\frac{g_l^2\Gamma\left(g_l^2\right)}{A_{0,l}\Gamma\left(1+g_l^2\right)}\left(\frac{\gamma_l}{\xi_l}\right)^{1/2}\middle|\begin{matrix}g_l^2\\ g_l^2-1,\quad 0\end{matrix}\right) \tag{11}$$

where γ_l is given by, [16]:

$$\gamma_l=\frac{\eta_l^2I_l^2}{2\sigma_l^2} \tag{12}$$

and ξ_l is the expected SNR at the *K-th* receiver, [16]:

$$\xi_l=\frac{\eta_l^2\ E\ I_l{}^{2}}{2\sigma_l^2} \tag{13}$$

with $E[I_l]$the expected value of the normalized irradiance I_l, which in the case of the studied channel is equal to:

$$E[I_l]=\frac{g_l^2\Gamma\left(g_l^2\right)}{\Gamma\left(1+g_l^2\right)} \tag{14}$$

III. AVERAGE BLER OF THE SIMO FSO SYSTEM

The estimation of the performance of the SIMO FSO system can be achieved by calculating the average block error rate (ABLER), which is the probability of more than M bit errors occurring in a block of N bits. This criterion constitutes a great indicator of the reliability of the system and the quality of the provided communication. Assuming that the system transmits the information in blocks of N bits, the probability of M errors bits reaching the receiver is expressed as,[20], [21]:

$$P(M,N;\gamma)=\sum_{m=M+1}^{N}\binom{N}{m}p^m(1-p)^{N-m} \tag{15}$$

where p is the probability of receiving a single error bit. For a SIMO system employing K receivers' diversity scheme with maximum ratio combining (MRC) and OOK modulation scheme, p is given by the following equation, [12]:

$$p=Q\left(\sqrt{\sum_{l=1}^{K}\gamma_l}\right) \tag{16}$$

The ABLER of the system can be extracted by averaging the BLER given in (15), which leads to the equation, [20]:

$$ABLER=\int_{\vec{\gamma}}P(M,N;\vec{\gamma})f_{\vec{\gamma}}(\vec{\gamma})d\vec{\gamma} \tag{17}$$

where $\vec{\gamma}=(\gamma_1,\gamma_2,...,\gamma_K)$ is the vector signal consisting of the K separate signals reaching the receiver. The substitution of (15) and (16) into (17) results in the following expression for the ABLER:

$$ABLER=\int_{\vec{\gamma}}\sum_{m=M+1}^{N}\binom{N}{m}Q\left(\sqrt{\sum_{l=1}^{K}\gamma_l}\right)^m\left[1-Q\left(\sqrt{\sum_{l=1}^{K}\gamma_l}\right)^{N-m}\right]f_{\vec{\gamma}}(\vec{\gamma})d\vec{\gamma} \tag{18}$$

In order to simplify the integral of (18) two steps are followed. Firstly, the approximation of the Q function given by, [22]

$$Q(x)\approx\frac{1}{12}\left[\exp\left(-\frac{x^2}{2}\right)+3\exp\left(-\frac{2x^2}{3}\right)\right] \tag{19}$$

is used, an approximation that is simple and very accurate, and then the binomial expansion and multinomial formulae are applied. Also, taking into account that the separate SNRs γ_l are independent, the ABLER of the system takes the form:

$$ABLER \approx \prod_{l=1}^{K}\left[\int_0^{\infty}\sum_{m=M+1}^{N}\binom{N}{m}\sum_{j=0}^{N-m}\binom{N-m}{j}\sum_{r=0}^{m+j}\binom{m+j}{r}\frac{(-1)^j}{12^{m+j-r}4^r}\times \right. \\ \left. \times\exp\left(-\frac{\gamma_l(3m+3j+r)}{12}\right)f_{\gamma_l}(\gamma_l)\,d\gamma_l\right] \qquad (20)$$

By substituting the combined PDF of the system (11) into (20) and the ABLER can be written as:

$$ABLER \approx \prod_{l=1}^{K}\left[\sum_{m=M+1}^{N}\binom{N}{m}\sum_{j=0}^{N-m}\binom{N-m}{j}\sum_{r=0}^{m+j}\binom{m+j}{r}\frac{(-1)^j}{12^{m+j-r}4^r}\frac{g_l^4\Gamma(g_l^2)}{A_{0,l}\Gamma(1+g_l^2)\sqrt{\xi_l}}\times \right. \\ \left. \times\int_0^{\infty}\gamma_l^{-1/2}\exp\left(-\frac{3m+3j+r}{12}\gamma_l\right)G_{1,2}^{2,0}\left(\frac{g_l^2\Gamma(g_l^2)}{A_{0,l}\Gamma(1+g_l^2)\sqrt{\xi_l}}\sqrt{\gamma_l}\,\middle|\,\begin{matrix} g_l^2 \\ g_l^2-1, \quad 0\end{matrix}\right)d\gamma_l\right] \qquad (21)$$

The final form of the ABLER can be obtained by solving the integral of (21) and is given by the following formula:

$$ABLER \approx \prod_{l=1}^{K}\left[\sum_{m=M+1}^{N}\binom{N}{m}\sum_{j=0}^{N-m}\binom{N-m}{j}\sum_{r=0}^{m+j}\binom{m+j}{r}\frac{(-1)^j}{12^{m+j-r}4^r}\times \right. \\ \times\frac{g_l^4\Gamma(g_l^2)}{4\sqrt{\pi\xi}A_{0,l}\Gamma(1+g_l^2)}\left(\frac{12}{3m+3j+r}\right)^{1/2}\times \\ \left. \times G_{3,4}^{4,1}\left(\left(\frac{g_l^2\Gamma(g_l^2)}{A_{0,l}\Gamma(1+g_l^2)}\right)^2\frac{3}{(3m+3j+r)\xi}\,\middle|\,\begin{matrix}\frac{1}{2}, & \frac{g_l^2}{2}, & \frac{g_l^2+1}{2} \\ \frac{g_l^2-1}{2}, & \frac{g_l^2}{2}, & 0, & \frac{1}{2}\end{matrix}\right)\right] \qquad (22)$$

IV. NUMERICAL RESULTS

In this section the average BLER of an FSO system with multiple receivers over saturated turbulence strength and pointing errors will be estimated using equation (22). We assume that the bit blocks consist of N=4 or 5 bits and a block will be considered erroneous in case more than 2 or 3 bits are faulty. The main parameters of the FSO link are presented in the following table:

Parameter	Value
ratio equivalent beam and displacement standard deviation (g)	2.52
Fraction of collected beam power (A_0)	0.039

TABLE 1: Parameters of the FSO link

The results will be extracted in case of a SISO system, K=1 and for the case of SIMO scheme, two receivers will be used, K=2. For simplicity, it can be assumed that the instantaneous SNR at the receivers are the same equal, i.e $\gamma_1 = \gamma_2 = ... = \gamma_K$.

In Figure 1, the average BLER results are presented for a SISO system. It is clear that as the number of acceptable erroneous bits in a certain number of bit blocks increases, the system has better performance as the block error probability decreases. This improvement can be more than one order of magnitude.

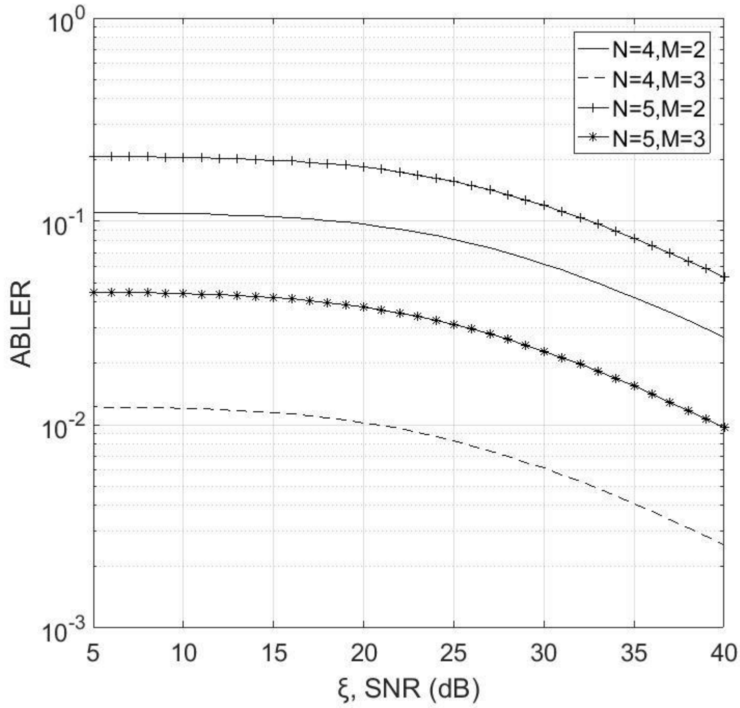

FIGURE 1: ABLER versus the expected SNR, ξ, for saturated turbulence and pointing errors for a SISO FSO system.

In Figure 2, the corresponding results for a SIMO system with two receivers are presented. Similarly to Figure 1, the BLER improves as the number of accepted erroneous bits in one block increases.

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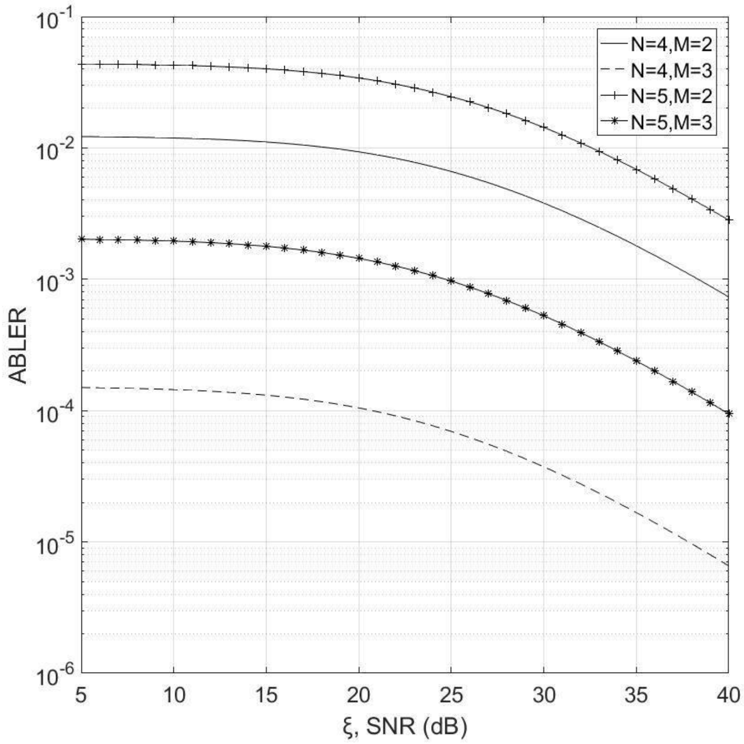

FIGURE 2: ABLER versus the expected SNR for saturated turbulence and pointing errors for a SIMO FSO system.

Comparing Figures 1 and 2 we can notice the remarkable improvement of system's performance as the number of receivers increases. This technique can achieve more than 2 orders of magnitudes lower BLER. More specifically, the percentage of the reduction in the BLER achieved by the SIMO system is presented in Figure 3, as a function of the expected SNR. So, even under strong turbulence conditions with pointing errors, the FSO link can perform reliably and securely satisfying even the most demanding links in modern networks.

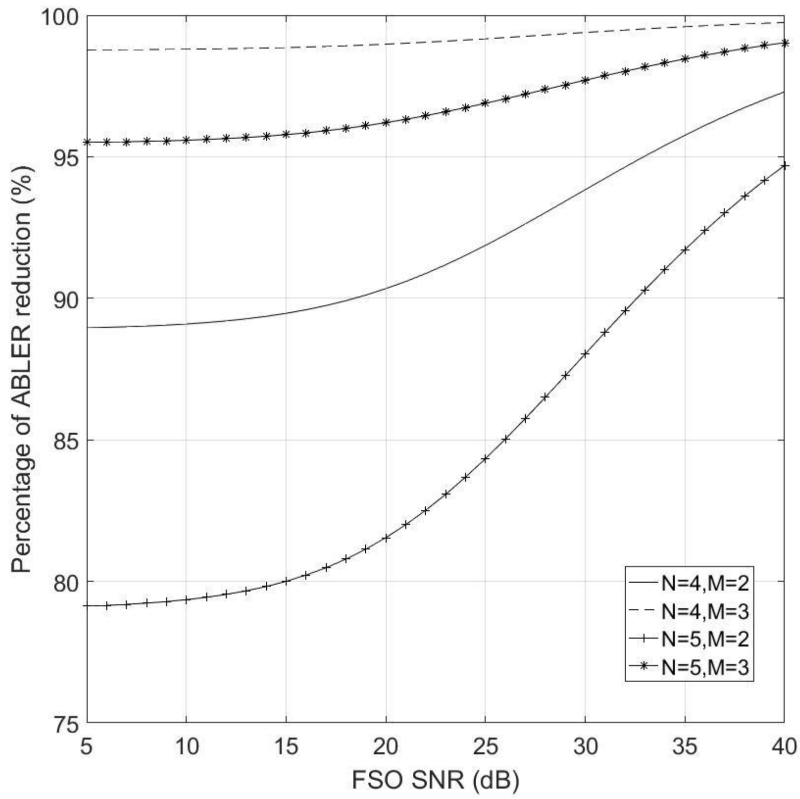

FIGURE 3: Percentage of the ABLER reduction achieved by the SIMO system compared to the SISO one, as a function of the expected SNR.

V. CONCLUSION

In this work, the performance of a SIMO FSO link under strong turbulence with pointing errors was investigated. a new mathematical expression for the average block error rate was derived, a metric that is very useful in order to implement a modern network. according to the results that were extracted, SIMO technique can further increase the performance and the reliability of an FSO system, providing high quality of service even under severe atmospheric conditions.

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